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HOC Simulation of Double-Diffusive Natural Convection in a Rectangular Cavity

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Abstract

Over the last few decades, Higher Order Compact (HOC) finite difference schemes have been tremendously gaining popularity among the researchers of computational fluid dynamics community, mainly because of their higher accuracies on smaller stencils. However, in most cases, their application is mainly confined to the computation of fluid flows and may be, in some cases to the problems of heat transfer only. The present work therefore intends to propose a new HOC scheme for problems in heat and mass transfer by extending an existing HOC scheme to a problem of combined heat and mass transfer. It is very interesting to observe that on a very coarser grid of size only 21X41 the results obtained are in very excellent match with the established ones.

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Keywords: Higher order compact; finite difference; computational fluid dynamics; stencil; heat transfer; mass transfer.

1. Introduction

Natural convection plays a major role in bulk of the industrial processes. Natural cooling or heating of substances is very crucial in detection of the fluid-phase composition and temperature at the phase interfaces during crystal growth. It is the foundation in modern electronics for the production of rods of lasers, microwave equipments, micro chips, transformers etc. Combination of concentration and temperature gradients in fluid flows in cavities lead to buoyancy-driven cavity flows. When transfer of mass and heat occur concurrently, double-diffusive natural convection takes place. Some of its applications include disposing nuclear waste, storing of energy in solar ponds, installation of grains, migrating moisture in fibrous insulators, transport of contaminants in civil engineering, detecting pollution of groundwater etc.

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In the present study, we extend an existing HOC scheme [1] for simulating problems of combined heat and mass transfer. We consider a problem of double-diffusive natural convection in a rectangular cavity of aspect ratio 2 as shown in Fig. 1. The rectangular cavity has height (H) twice of the length (L) of the cavity. The top and bottom walls of the cavity are assumed to be adiabatic with no mass flux while the vertical walls are differentially heated and concentrated. A hot temperature (T_h) and a cold temperature (T_c) are uniformly imposed along the left and right walls of the cavity respectively. Moreover, the left wall has a higher concentration (source) as compared to the right wall (sink). As time progresses, the fluid mixture diffuses from the source to the sink. For Rayleigh number ($Ra = 10^5$), the flow is investigated for different values of Hartmann numbers (Ha) and buoyancy ratio (N) and results obtained are compared with the available results in the literature [2, 3].

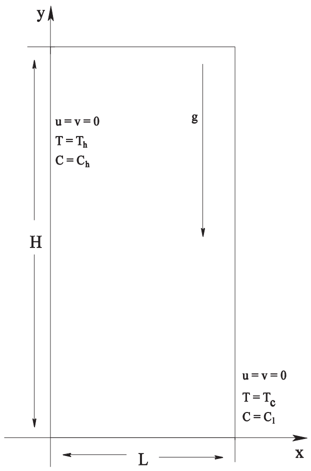


Fig. 1. Problem Configuration.

Nomenclature

H	height of the cavity
L	length of the cavity
T_h	hot temperature at left wall
T_c	cold temperature at right wall
C_h	high concentration at left wall
C_l	low concentration at right wall
u	horizontal component of velocity
v	vertical component of velocity
Ra	thermal Rayleigh number
Ha	Hartmann number
N	buoyancy ratio
Pr	Prandtl number
Le	Lewis number
φ	dimensionless heat generation or absorption coefficient
g	acceleration due to gravity
ψ	stream function
ω	vorticity
T	temperature
C	concentration

In the problem set up (Fig. 1), the rectangular cavity is filled with a Boussinesq fluid of gaseous mixture. The flow inside the cavity is assumed to be incompressible, viscous, Newtonian, heat absorbing or generating and electrically conducting. Owing to the temperature and concentration differences between the vertical walls of the cavity, double diffusive natural convection takes place under the influence of body force (gravity *g*).

1.1. Governing Equations

The equations governing the flow inside the cavity may be written in non-dimensional form as

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -\nabla^2 \psi \tag{1}$$

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = Ra\,\mathrm{Pr}\left(-\frac{\partial T}{\partial x} + N\frac{\partial C}{\partial x}\right) + \mathrm{Pr}\,\nabla^2\omega - \mathrm{H}a^2\mathrm{Pr}\frac{\partial v}{\partial x} \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \nabla^2 T + \varphi(T + 0.5) \tag{3}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \left(\frac{1}{Le}\right)\nabla^2 C \tag{4}$$

where ω is the vorticity, u and v are the horizontal and vertical component of the velocity vector, ψ is the stream function, *Ra* is the Rayleigh number, *Pr* is the Prandtl number, *T* is the temperature, *N* is the buoyancy ratio, *C* is the concentration, *Ha* is the Hartmann number, φ is the dimensionless heat generation or absorption coefficient, Le is the Lewis number and *t* is the non-dimensional time.

The initial conditions (at time, $t = 0$) for the problem are:

$$u = v = \psi = 0,\ T = -0.5,\ C = -0.5,\ \forall x, y.$$

The boundary conditions for the problem are:

$$\text{at } x = 0, \quad u = v = \psi = 0,\ \omega = -\frac{\partial^2 \psi}{\partial x^2},\ T = 0.5, \quad C = 0.5, \quad \forall y.$$

$$\text{at } x = 1, \quad u = v = \psi = 0,\ \omega = -\frac{\partial^2 \psi}{\partial x^2},\ T = -0.5,\ C = -0.5,\ \forall y.$$

$$\text{at } y = 0, \quad u = v = \psi = 0,\ \omega = -\frac{\partial^2 \psi}{\partial y^2},\ \frac{\partial T}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0, \quad \forall x.$$

$$\text{at } y = H/L,\ u = v = \psi = 0,\ \omega = -\frac{\partial^2 \psi}{\partial y^2},\ \frac{\partial T}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0, \quad \forall x.$$

2. Method of Analysis

To develop the new HOC scheme (extended), we consider the (9, 5) HOC scheme developed by Kalita and Chhabra [1] and reconstruct each of the governing equations (equations (1) - (4)) for the problem under consideration in such a way that they fit into the HOC scheme [1]. The resultant scheme is of fourth order spatial and second order temporal accuracy.

Kalita and Chhabra [1] considered the 2-D second-order unsteady partial differential equation (p. d. e.) given by

$$a\frac{\partial\phi}{\partial t}-\nabla^2\phi+c(x,y,t)\frac{\partial\phi}{\partial x}+d(x,y,t)\frac{\partial\phi}{\partial y}+\varepsilon(\mathrm{x},\mathrm{y},\mathrm{t})\phi=g(x,y,t) \tag{5}$$

where a is a constant, ϕ is a transport variable, $c(x, y, t)$ and $d(x, y, t)$ are the convective coefficients, $\varepsilon(x, y, t)$ is the reaction coefficient, ∇^2 is the Laplacian operator and $g(x, y, t)$ is the forcing function.

They considered uniform meshes of steps h and k along x- and y-directions and took the standard central difference approximation of the p. d. e. (5). To approximate the higher derivatives in the truncation error, they considered this original p. d. e. as an auxiliary equation and obtained a HOC (fourth order spatial and second order temporal) approximation of Eq. (5) on a (9, 5) stencil as

$$\begin{aligned}&a\left[1+\left(\frac{h^2}{12}-\frac{\Delta t}{2a}\right)(\delta_x^2-c_{ij}\delta_x)+\left(\frac{k^2}{12}-\frac{\Delta t}{2a}\right)(\delta_y^2-d_{ij}\delta_y)+\frac{\Delta t}{2a}\varepsilon_{ij}\right]\delta_t^+\phi_{ij}+\\&\left(-\alpha_{ij}\delta_x^2-\beta_{ij}\delta_y^2+C_{ij}\delta_x+D_{ij}\delta_y+E_{ij}\right)\phi_{ij}-\frac{\left(h^2+k^2\right)}{12}\left(\delta_x^2\delta_y^2-c_{ij}\delta_x\delta_y^2-d_{ij}\delta_x^2\delta_y-\gamma_{ij}\delta_x\delta_y\right)\phi_{ij}=G_{ij}\end{aligned} \tag{6}$$

where ϕ_{ij} represents $\phi(x_i, y_j)$; δ_x and δ_y are the first-order central difference operators and δ_x^2 and δ_y^2 are the second-order central difference operators, δ_t^+ is the first-order temporal forward difference operator, Δt is the uniform time-step and the coefficients α_{ij}, β_{ij}, C_{ij}, D_{ij}, E_{ij}, G_{ij}, γ_{ij} are given by:

$$\alpha_{ij}=1+\frac{h^2}{12}\left(c_{ij}^2-2\delta_x c_{ij}-\varepsilon_{ij}\right)$$

$$\beta_{ij}=1+\frac{k^2}{12}\left(d_{ij}^2-2\delta_y d_{ij}-\varepsilon_{ij}\right)$$

$$C_{ij}=\left[1+\frac{h^2}{12}\left(\delta_x^2-c_{ij}\delta_x\right)+\frac{k^2}{12}\left(\delta_y^2-d_{ij}\delta_y\right)+\frac{\Delta t}{2}\delta_t^-\right]c_{ij}+\frac{h^2}{12}\left(2\delta_x-c_{ij}\right)\varepsilon_{ij}$$

$$D_{ij}=\left[1+\frac{h^2}{12}\left(\delta_x^2-c_{ij}\delta_x\right)+\frac{k^2}{12}\left(\delta_y^2-d_{ij}\delta_y\right)+\frac{\Delta t}{2}\delta_t^-\right]d_{ij}+\frac{k^2}{12}\left(2\delta_y-d_{ij}\right)\varepsilon_{ij}$$

$$E_{ij}=\left[1+\frac{h^2}{12}\left(\delta_x^2-c_{ij}\delta_x\right)+\frac{k^2}{12}\left(\delta_y^2-d_{ij}\delta_y\right)+\frac{\Delta t}{2}\delta_t^-\right]\varepsilon_{ij}$$

$$G_{ij}=\left[1+\frac{h^2}{12}\left(\delta_x^2-c_{ij}\delta_x\right)+\frac{k^2}{12}\left(\delta_y^2-d_{ij}\delta_y\right)+\frac{\Delta t}{2}\delta_t^-\right]g_{ij}$$

$$\gamma_{ij} = \frac{2}{h^2 + k^2}(h^2 \delta_x d_{ij} - k^2 \delta_y c_{ij}) - c_{ij} d_{ij}$$

where δ_t^- denotes the first-order backward difference operator for time. Throughout the derivation, it is assumed that the forcing function g and all its derivatives are either known analytically or their discrete approximations are known. To implement this scheme in equation (1), we consider the steady-state version of the HOC scheme [1] given by

$$\left(-\alpha_{ij}\delta_x^2 - \beta_{ij}\delta_y^2 + C_{ij}\delta_x + D_{ij}\delta_y + E_{ij}\right)\phi_{ij} - \frac{\left(h^2 + k^2\right)}{12}\left(\delta_x^2\delta_y^2 - c_{ij}\delta_x\delta_y^2 - d_{ij}\delta_x^2\delta_y - \gamma_{ij}\delta_x\delta_y\right)\phi_{ij} = G_{ij} \tag{7}$$

where α_{ij}, β_{ij}, C_{ij}, D_{ij}, E_{ij}, G_{ij}, γ_{ij} are the same as was in the unsteady formulation but with $\Delta t = 0$.
To solve (1), we consider ϕ as ψ and $c = d = \varepsilon = 0$ and $g = \omega$.
For the vorticity equation (2), we replace a, ϕ, c, d, ε and g in equation (5) by 1, ω, u/Pr, v/Pr, 0 and $Ra\left(-\frac{\partial T}{\partial x} + N\frac{\partial C}{\partial x}\right) - \mathrm{H}a^2\frac{\partial v}{\partial x}$ respectively. The source term in the vorticity transport equation is not known explicitly and is in derivative form. In order to approximate this source term we use the standard second order central difference scheme.
Next, to solve equation (3), we replace a, ϕ, c, d, ε and g in equation (5) by 1, T, u, v, $-\varphi$ and 0.5φ respectively.
Finally, for equation (4), we replace a, ϕ, c, d, ε and g in equation (5) by Le, C, Leu, Lev, 0 and 0 respectively.

2.1. Boundary conditions

For an mxn grid, we have the following boundary conditions:
The stream function ψ equals to zero on all the boundaries, i.e. $\psi_{i,j} = 0$, for $i \le 1 \le m, j \le 1 \le n$.
For vorticity boundary conditions, Jensen's formula [4] is used.
The boundary conditions for temperature and concentration at the left and right boundaries are Dirichlet boundary conditions (see section 1). The top and bottom boundary conditions for both are Neumann, where we have employed first order forward difference approximations.

Now, to solve the system (1)-(4), we adopt an iterative procedure and employ a decoupled algorithm. We solve all the state variables, namely, stream-function, vorticity, temperature and concentration in a sequential manner but separately. At each time iterates, we have over-relaxed both the temperature and concentration while the stream-function and vorticity have been slightly under-relaxed. The coefficient matrices arising from the HOC discretization of the governing equations (1)-(4) are not diagonally dominant and therefore conventional iterative solvers like Gauss-Seidel cannot be employed. As such, we have utilized the Bi-Conjugate gradient stabilized (BiCG stab) method without any preconditioning [5].

3. Results

We perform numerical simulations on a HCL Desktop PC with corei5@3ghz processor and 4gb ddr3 ram. For all the computations we chose Pr = 1.0, $Ra = 10^5$ on a uniform grid of size 21x41with time-step 0.0001. In order to validate the current HOC method, we compare our results with the benchmark solution of Ma [2] and Nishimura *et al.* [3] which gives us an excellent comparison. For different values of the parameters, *Ra, Pr, N, Ha,* φ and *Le,* our simulated results are shown in Figs. (2)-(6). Meanwhile, it is noteworthy that our results produce very good results on a much coarser grid of size 21x41 only, as opposed to the earlier results [2, 3] which were obtained on comparatively larger grids.

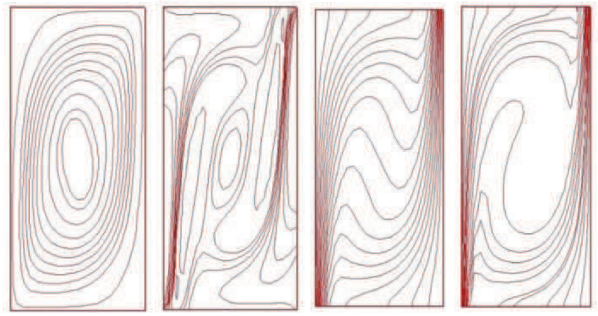


Fig. 2. Stream lines, vorticity contours, isothermal lines and isoconcentration lines, for Ha = 0, Pr = 1.0, Le = 2.0, Ra = 10^5, N = 0.8, φ = 0.0 at non-dimensional time 1.75.

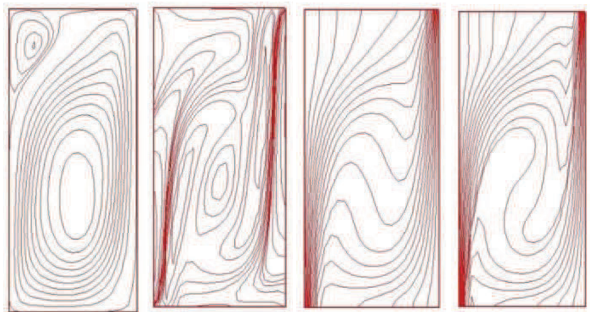


Fig. 3. Stream lines, vorticity contours, isothermal lines and isoconcentration lines, for Ha = 0, Pr = 1.0, Le = 2.0, Ra = 10^5, N = 0.8, φ = 0.0 at non-dimensional time 1.90.

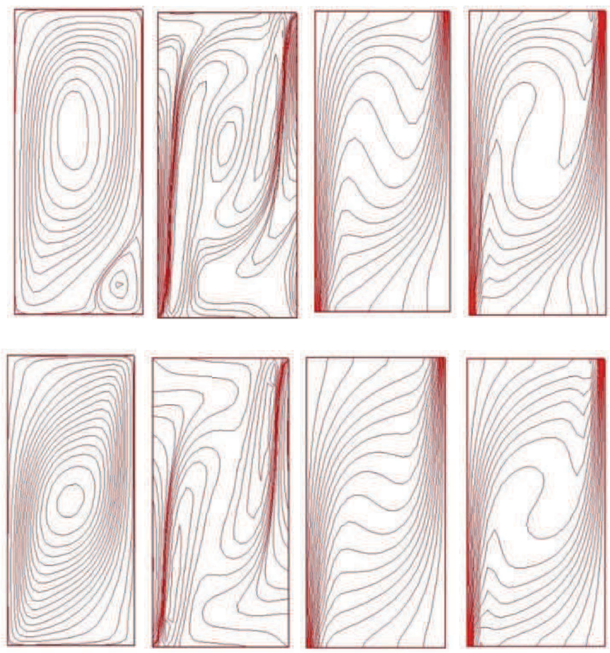


Fig. 4. Stream lines, vorticity contours, isothermal lines and isoconcentration lines, for Pr = 1.0, Le = 2.0, Ra = 10^5, N = 0.8, φ = -1.0 at non-dimensional time 1.75: Top Row (Ha = 0), Bottom Row (Ha = 10).

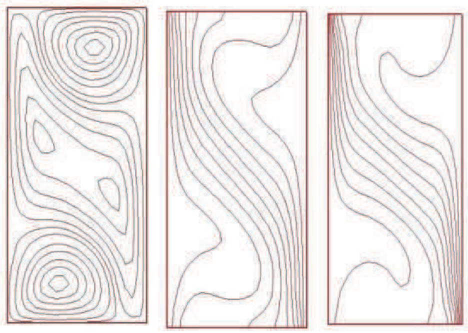
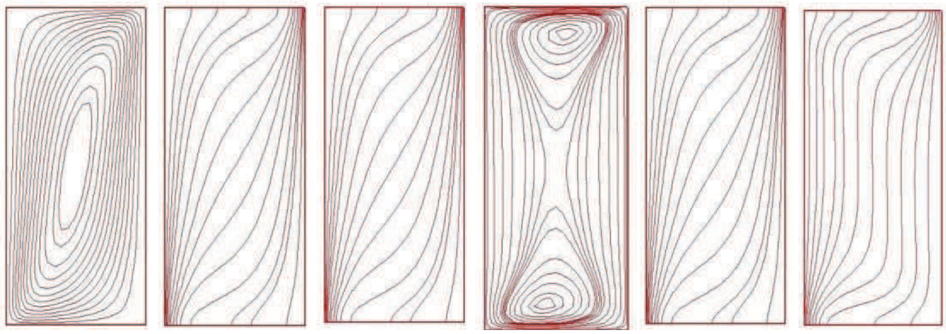


Fig. 5. Stream lines, isothermal lines and isoconcentration lines, for Ha = 0, Pr = 1.0, Le = 2.0, Ra = 10^5, N = 1.3, φ = 0.0 at non-dimensional time 1.75.



(a) (b) (c) (d) (e) (f)

Fig. 6: Stream lines, isothermal lines and isoconcentration lines, for Pr = 1.0, Le = 2.0, Ra = 10^5, N = 0.8, φ = 0.0 at time = 1.75: Ha = 25 (figures a, b and c respectively), Ha = 50 (figures d, e and f respectively).

4. Conclusion

The work in the current manuscript is concerned with the HOC computation of the double diffusive natural convection in a rectangular cavity of grid aspect ratio 2 and with a magnetic field and heat source. The horizontal walls of the cavity are adiabatic and the vertical walls are differentially heated. Moreover, the left wall has a higher concentration than the right wall. This is probably the first time that an HOC method is proposed for the study of this problem. The solution strategy employed in this manuscript has the potential of being extended to other similar physical problems like the primitive variable formulation of the Navier-Stokes equations and other intriguing fluid flow phenomena governed by the 2D Navier-Stokes equations. At present, efforts are going on to study this problem in a inclined rectangular cavity.

References

[1] J. C. Kalita, P. Chhabra, An improved (9, 5) higher order compact scheme for the transient two-dimensional convection-diffusion equation, Int. J. Numer. Meth. Fluids. 51 (2006) 703–717.

[2] C. Ma, Lattice BGK simulations of double diffusive natural convection in a rectangular enclosure in the presences of magnetic field and heat source. Nonlin. Anal.: Real World Appl. 10 (2009) 2666–2678.

[3] T. Nishimura, M. Wakamatsu, A. M. Morega, Oscillatory double-diffusive convection in a rectangular enclosure with combined horizontal temperature and concentration gradients, Int. J. Heat Mass Transfer. 41 (1998) 1601–1611.

[4] P. Roache, Computational Fluid Dynamics, revised ed., Hermosa, Albuquerque, NM, 1982.

[5] C. T. Kelley, Iterative Methods for Linear and Nonlinear Equations, SIAM Publications, Philadelphia, Pennsylvania, 1995.